Validating Sparse Acoustic Localizations: Two-Stage Error Evaluation

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Executive Summary

When tracking marine mammals using a sparse acoustic array, one of the primary challenges is distinguishing true physical locations from algorithmic "phantom" positions. To guarantee the accuracy of our data, the SonarPoint processing pipeline employs a strict two-stage error validation process: **Time Difference of Arrival (TDOA) Closure Error (Stage 5)** followed by **Minimum Residual Error evaluation (Stage 6)**.

We perform both evaluations because they diagnose fundamentally different failure points. Stage 5 acts as an *acoustic reality check*, verifying that the signal processing successfully tracked the exact same sound wavefront from a single sound event as it passed across all sensors. Stage 6 acts as a *geometric reality check*, ensuring that those tracked time delays map to a physically possible 3D coordinate in the ocean. Relying on only one method leaves the root cause of spatial errors ambiguous. By evaluating both stages sequentially, we can confidently isolate true animal locations, discard phantom algorithmic coordinates, and immediately pinpoint the source of any systemic errors (e.g., multipath interference vs. incorrect mooring surveys). Because the two-stage validation is independent for each localization, it is in particular powerful for isolated cetacean calls where TDOA brushing based on a series of consistent TDOA is not available.

Critical Distinction: Detection vs. Localization

It is important to emphasize that this method validates the location of the sound source, not the existence of the sound itself. A high closure error does not mean the acoustic detection was a false positive (i.e., it does not mean there was no animal). It simply means that the cross-correlation algorithm failed to track the exact same acoustic wave path across all four moorings (often due to multipath interference or background noise). The animal is real, but the mathematical coordinate generated for it is a phantom. Detection validation must be done at stage 4 prior to the stage 5 and 6 localization validations.

The TDOA Closure Principle (Stage 5)

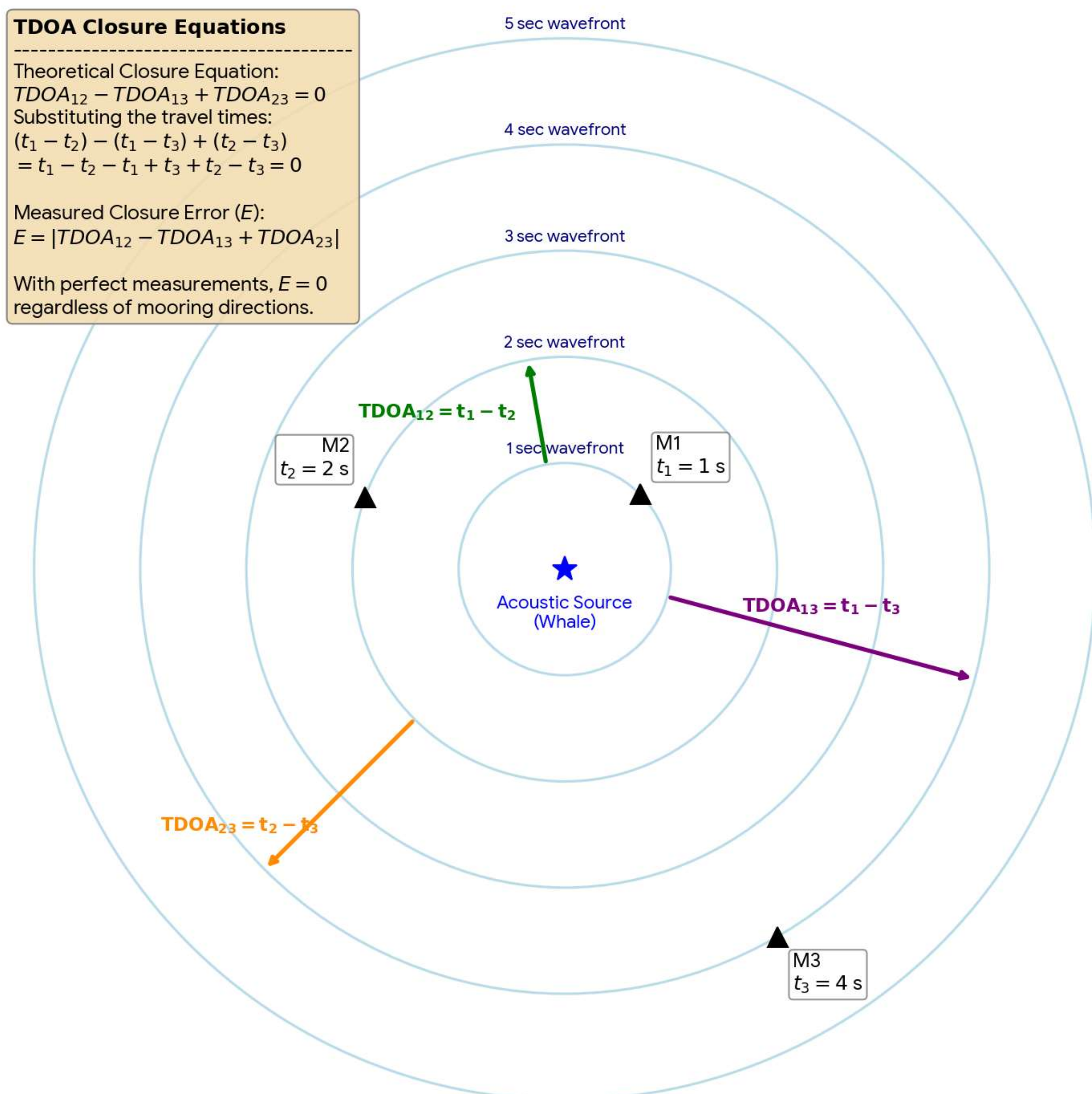
In a sensor network, a single sound wave emitted from a physical source will reach different hydrophones at different absolute times (t1, t2, t3). While we cannot know the absolute emission time, cross-correlation allows us to measure the Time Difference of Arrival (TDOA) between pairs of moorings (e.g., TDOA12 = t1 - t2).
Because these time differences are derived from the exact same physical event, they must mathematically form closed geometric loops. Assume mooring M1 is closest to a sound source, mooring M2 further away in any direction and mooring M3 furthest from the source. The wavefront of the sound will reach M1 first, then M2 and finally M3. If you add the delay between Mooring 1 and Mooring 2 to the delay between Mooring 2 and Mooring 3, the sum must perfectly equal the direct delay between Mooring 1 and Mooring 3. This can be generalized independent of the mooring geometry or distance sequence as follows.

Theoretical Equation:
TDOA12 + TDOA23 = (t1 - t2) + (t2 - t3)
TDOA12 + TDOA23 = t1 - t3
TDOA12 + TDOA23 = TDOA13

The Closure Error (E):
In reality, if the algorithm accidentally locks onto a surface-reflected echo on one mooring and a direct path on another, or if it catches a random noise spike, the measured delays will contradict each other. The absolute difference between the loop components is the Closure Error:

E = | TDOA12 + TDOA23 - TDOA13 |

If E is near zero, the acoustic event is definitively a single, trackable physical wavefront originating at the location of the sound source. If E is high, the resulting X/Y position is a phantom.



The Two-Stage Validation: Why Both are Required

While it might seem redundant to evaluate error twice, Stage 5 and Stage 6 diagnose fundamentally different failure points in the system.

- **Stage 5: TDOA Closure (Signal Validation):** TDOA closure evaluates the internal consistency of the signal processing before any spatial coordinate is ever calculated. It guarantees that the cross-correlation algorithm successfully tracked the exact same acoustic wave path across the moorings. Because this calculation is independent of mooring geometry, the closure loop will balance to zero regardless of where the moorings are located in space. However, Stage 5 has a specific blind spot: if the cross-correlation algorithm locks onto a surface-reflected path on *all* moorings, the TDOA loop will still perfectly close (resulting in an error near zero), giving false confidence to a delayed signal.

- **Stage 6: Minimum Residual Error (Geometric Validation):** Stage 6 evaluates the physical goodness-of-fit. It takes the assumed physical locations of the moorings and attempts to find a 3D coordinate in the ocean that perfectly matches the measured time delays. If all moorings locked onto a reflected path (which passed Stage 5), Stage 6 will catch the error because those universally delayed travel times will not mathematically fit anywhere within the real-world physical volume of the array. Stage 6 also catches environmental modeling errors (like uniform speed of sound errors) and incorrect mooring placements.

By performing both calculations sequentially, we can clearly identify the precise source of any location error, as outlined in the matrix below.

Error Identification Matrix

Potential Source of Error	**Stage 5: TDOA Closure Result**	**Stage 6: Minimum Residual Error Result**	**Diagnostic Resolution**
Mixed Multipath Interference (e.g., Direct path on M1, reflected on M2)	**HIGH ERROR:** Time delays contradict each other; loop fails to close.	**HIGH ERROR:** Disjointed delays cannot resolve to a single physical point.	Stage 5 immediately identifies a signal cross-correlation failure before mapping is attempted.
Uniform Multipath (Reflected path locked on *all* moorings)	**LOW ERROR:** The same physical wave path was tracked, so the TDOA loop	**HIGH ERROR:** The uniform delay makes the intersection physically impossible in the	Stage 6 catches the acoustic reality check that Stage 5 misses.

	perfectly closes.	3D volume.	
Mooring Location Errors (e.g., unrecorded drift, poor pinger survey)	**LOW ERROR:** TDOA closure math is completely independent of the physical mooring coordinates.	**HIGH ERROR:** Real-world sound travel times will not mathematically fit the incorrect 3D grid points.	Isolates the error entirely to the array deployment/survey data, clearing the algorithm of fault.
Uniform Speed of Sound Error (e.g., assuming 1500 m/s when true speed is 1480 m/s)	**LOW ERROR:** Stage 5 uses raw timestamps; the TDOA loop perfectly closes regardless of the actual sound speed.	**HIGH ERROR:** The incorrect speed causes the calculated distance radii to be universally too long or short, preventing a clean intersection.	Identifies systemic environmental modeling errors while confirming the signal processing itself was correct.
Shallow Hyperbola Intersection (Far-away source / GDOP)	**LOW ERROR:** Signal processing successfully tracked the true sound.	**LOW ERROR:** The mathematical fit is technically good, despite the geographic footprint being vast.	Neither stage fully captures this; requires a secondary geographic multiplier to estimate true spatial variance.
Valid, True Localization	**LOW ERROR:** Pristine mathematical closure.	**LOW ERROR:** Time delays perfectly fit a realistic 3D coordinate.	High-confidence position fix.

Conclusion

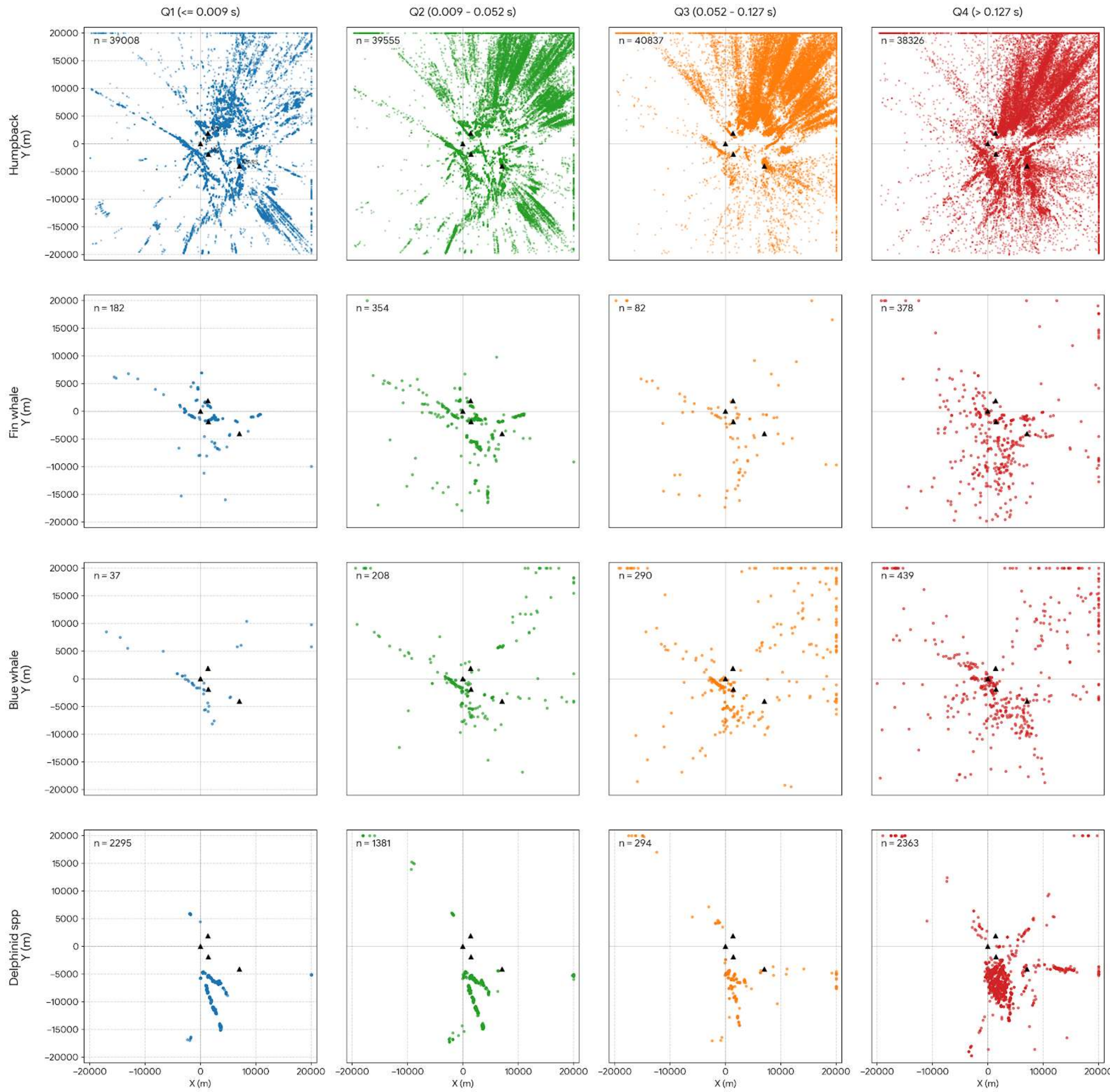
Relying solely on visual tracklines or assumed kinematic behaviors (like swim speed) to validate sparse array localizations is inherently limiting, especially for rarely vocalizing animals that do not produce enough calls to form a continuous track. The two-stage validation method solves this by allowing each individual position to be validated entirely on its own mathematical merit, independent of any prior or subsequent detections. By gating localizations using a hard closure error threshold alongside geometric residuals, researchers can objectively isolate true physical coordinates and confidently discard phantom positions on a ping-by-ping basis. Consequently, even a single, isolated call from a rarely vocalizing cetacean can be position-fixed with high confidence.

Appendix A: Empirical Dataset Evaluation

Species-Specific Array Performance (Closure Quartiles)

To evaluate array performance across different species in the LMR 1.3 deployment, the average closure error for every localized call was calculated across all four possible triangles in a four-mooring array. The data was then split into four statistical quartiles, keeping the error thresholds identical across all species for direct comparison.

Quartile	Closure Error Range	Interpretation
Q1 (Blue)	≤ 0.009 seconds	Pristine mathematical closure. Highly verified true physical locations. Individual animal tracks are clearly apparent.
Q2 (Green)	0.009 - 0.052 seconds	Minor timing drift (likely distant calls with slight multipath/noise).
Q3 (Orange)	0.052 - 0.127 seconds	Significant geometric instability. Prone to radial hyperbola smearing.
Q4 (Red)	> 0.127 seconds	Massive mathematical failure. Resulting X/Y coordinates are phantoms and appear as a diffuse cloud.

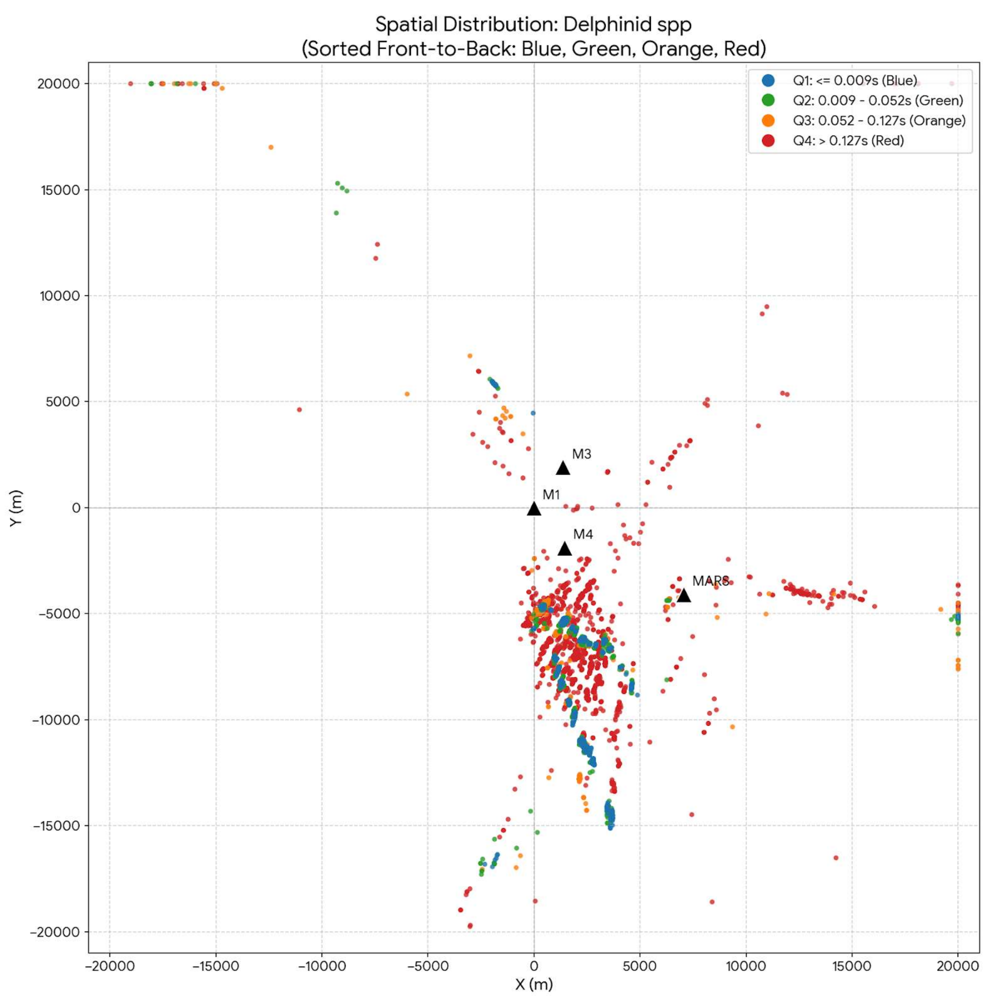


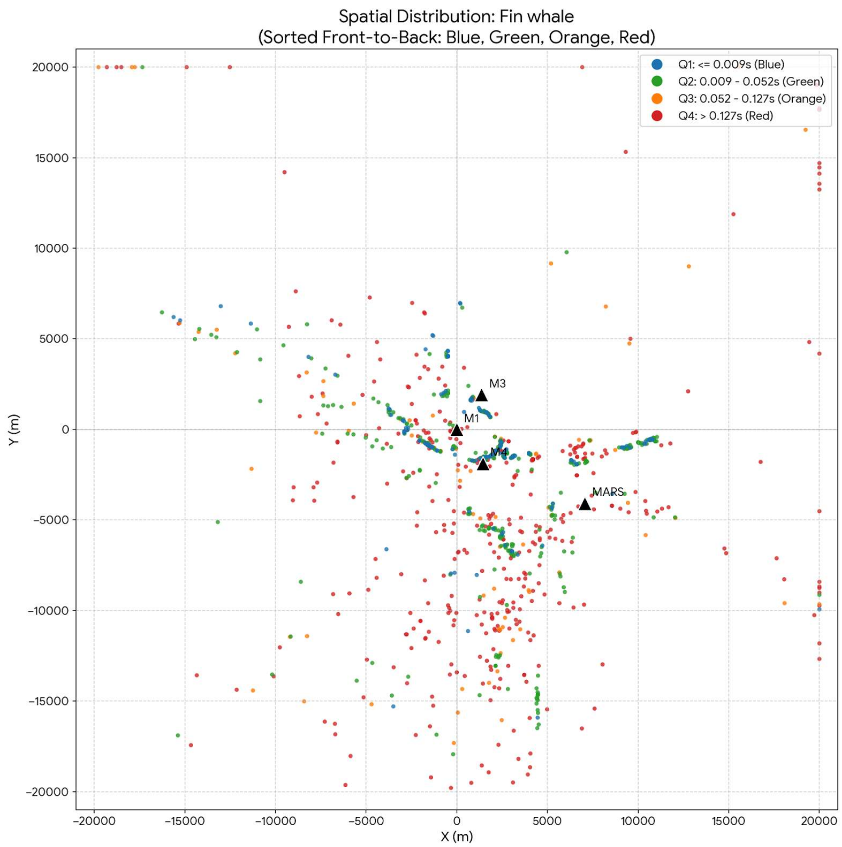
Analysis of Results by Species

- **Delphinid spp. and Deep-Water Multipath:** Over a third of Delphinid localizations fall into the worst closure error quartile (Q4), collapsing into a dense "phantom" cluster directly over the inner array. Because high-frequency Delphinid calls attenuate quickly, they are only localized at close ranges. At these close horizontal ranges, a deep mooring (e.g., 1000m) receives the direct path and a strong surface reflected path. The cross-correlation algorithm struggles to resolve this multipath interference, breaking the

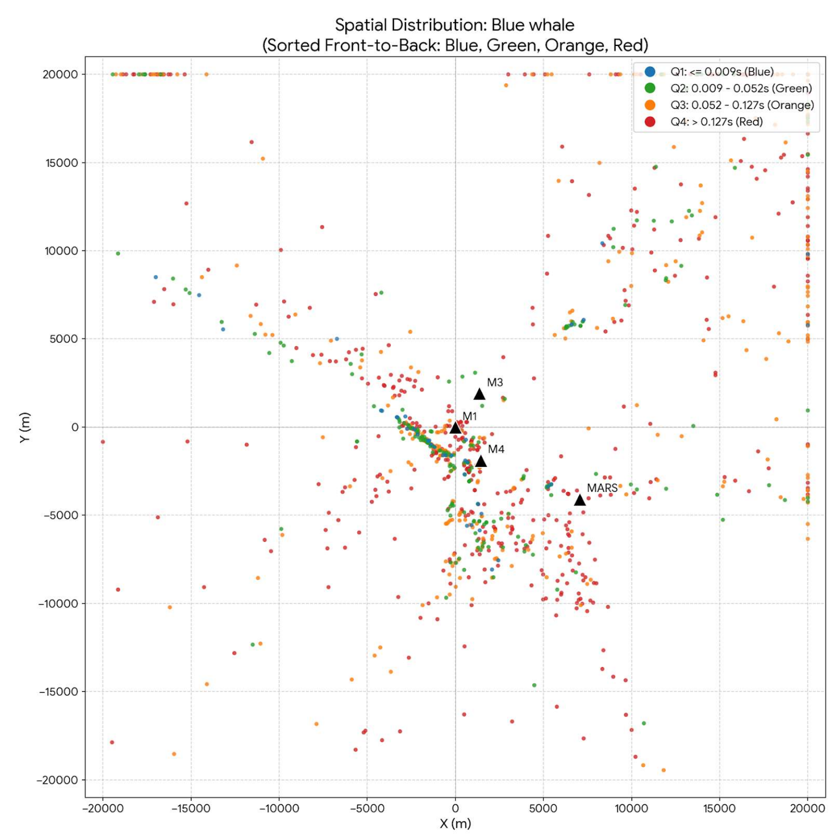
TDOA loop. However, when a clean path is captured (Q1), the resulting tracks are highly accurate.

- **Fin Whales:** The algorithm performs exceptionally well for Fin whales. A large portion of their detections land in Q1 and Q2. Their mathematically verified (Q1) tracklines cleanly traverse the array, largely avoiding the algorithmic collapse seen in other species.
- **Blue Whales:** Blue whales present a significant localization challenge for this array baseline. The vast majority of their mapped coordinates fall into the Q3 and Q4 high-error categories, with very few pristine Q1 tracks. This indicates the algorithm frequently struggles to align the extremely long wavelengths of Blue whale calls into perfect, unambiguous cross-correlation peaks across four separate channels.
- **Humpback Whales:** As the most populous dataset, Humpbacks clearly display the dual nature of spatial errors. Applying the Q1/Q2 filter isolates true, linear transit movements. Filtering out Q3 and Q4 successfully removes both the central multipath collapses (red center dots) and the geometric "radial smearing" (spokes stretching outwards), where tiny timing errors at long distances warp coordinates by thousands of meters.

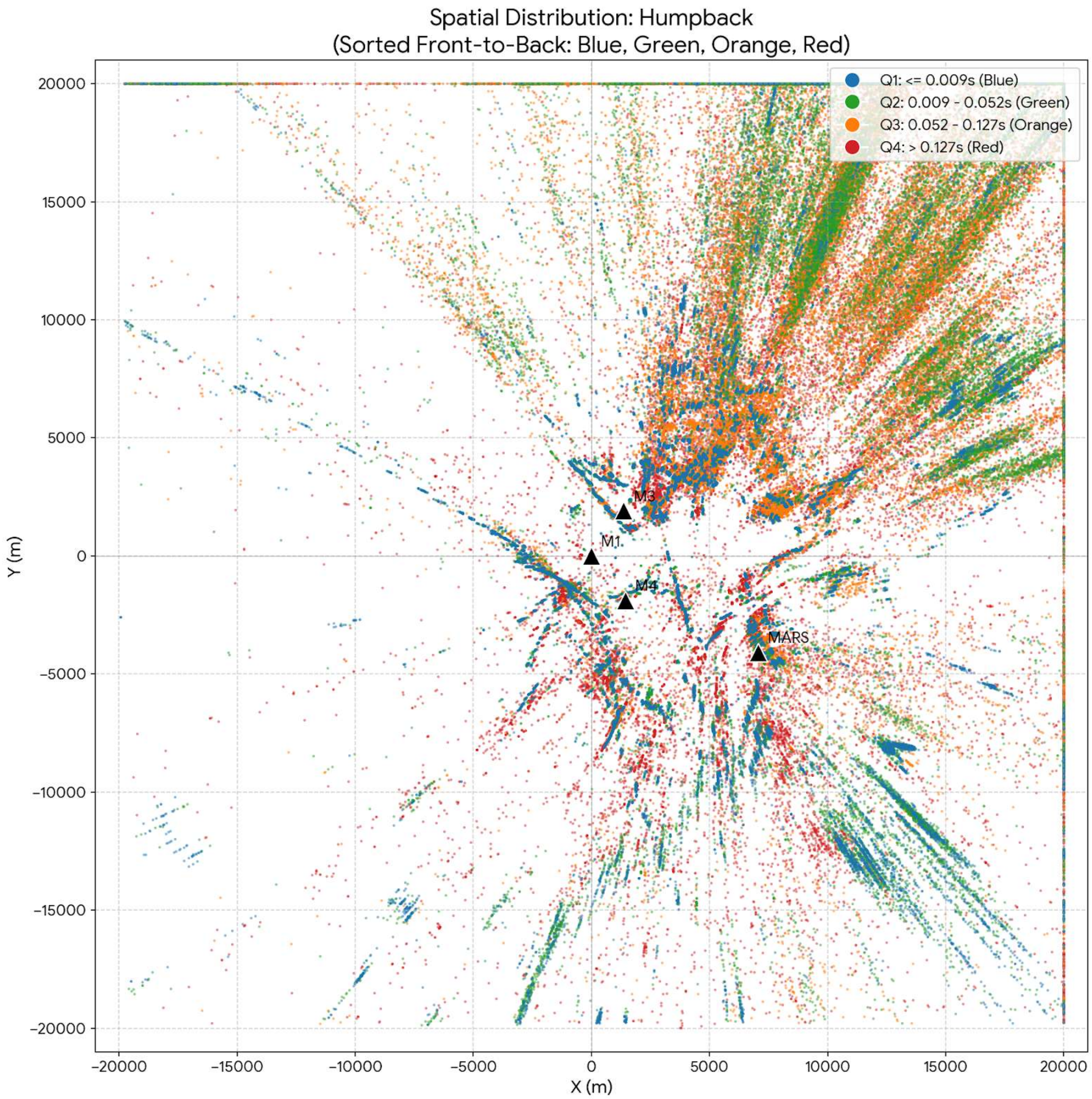




Spatial Distribution: Fin whale
(Sorted Front-to-Back: Blue, Green, Orange, Red)
Q1: <= 0.009s (Blue)
Q2: 0.009 - 0.052s (Green)
Q3: 0.052 - 0.127s (Orange)
Q4: > 0.127s (Red)
M3
M1
M4
MARS
Y (m)
X (m)



Spatial Distribution: Blue whale
(Sorted Front-to-Back: Blue, Green, Orange, Red)
Q1: <= 0.009s (Blue)
Q2: 0.009 - 0.052s (Green)
Q3: 0.052 - 0.127s (Orange)
Q4: > 0.127s (Red)
M3
M1
M4
MARS
Y (m)
X (m)


Spatial Distribution: Humpback
(Sorted Front-to-Back: Blue, Green, Orange, Red)
Q1: <= 0.009s (Blue)
Q2: 0.009 - 0.052s (Green)
Q3: 0.052 - 0.127s (Orange)
Q4: > 0.127s (Red)
M3
M1
M4
MARS
Y (m)
X (m)
20000
15000
10000
5000
0
−5000
−10000
−15000
−20000